

Capstan Equation

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- Convert degrees of surface contact to radian (360 degrees = 6.28319 Radians, equivalent to $2 \times \pi$)
- Coefficient of friction x radian = FACTOR
- e constant exponential by FACTOR above
- Result gives ratio of load to 1

Example: A rope goes around a pole (any diameter) and makes 315 degrees of contact. 315 degrees equals 5.4978 radian (or 1.75π). Coefficient of friction between the rope and the pole is determined to be .25

Example Equation:

$$.25 \text{ (friction)} \times 5.4978 \text{ (radian)} = 1.37445 \text{ (Factor)}$$

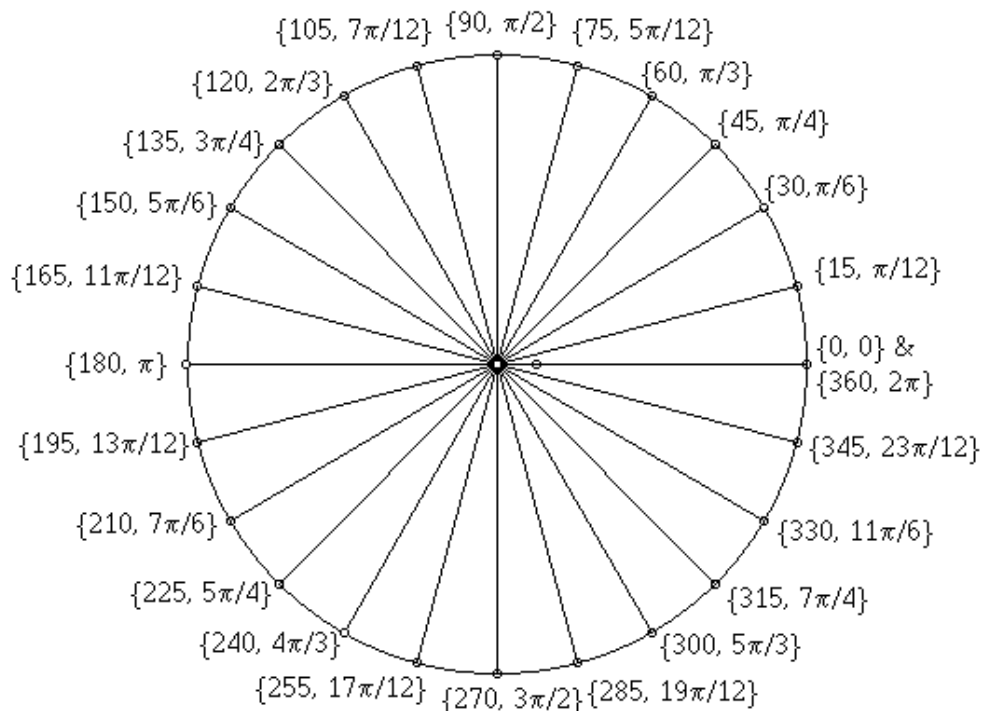
$$e^{1.37445} = \mathbf{3.952:1} \text{ Ratio (1 pound of force will hold 3.952 pounds of weight)}$$

On scientific calculator press e^x (Exponential Constant Function), enter friction value in decimal \times rad π , press = (to get rad, divide the degree of contact by 180. 360 degrees = 2, 270 degrees = 1.5, etc.)

The below coefficient Table 1 is from the paper *Rescue Systems Mechanics, Interim Report* by Timothy Manning. The author of this paper has not verified these values. See the author's own test results below.

Table 1. Static coefficients of friction for nylon kernmantle rope.

sandstone	limestone	granite	Cordura nylon	Canvas	Stainless steel	Galvanized steel	Aluminum	HDPE	Wood
0.6-0.8	0.5-0.9	0.5-0.9	0.43	0.57	0.7	0.6	0.7	0.13-0.25	0.5



Non-scientific testing by the author:

Test Bar: 1.0" Diameter Stainless Steel Polished Rack Bar					
Cordage / Rope	Weight Supported (in pounds)	2 Wrap Holding Force In Pounds	1 Wrap Holding Force In Pounds	Ratio of holding force to weight	Calculated Coefficient of Friction
5.9mm PowerCord	10.1		3.2	3.156	.18 / .19
5.9mm PowerCord	10.1	1.3		7.769	.16 / .17
9mm Sterling HTP	10.3		5.4	1.907	.10 / .11

Test Bar: 2.25" Diameter Aluminum Tube					
Cordage / Rope	Weight Supported (in pounds)	2 Wrap Holding Force In Pounds	1 Wrap Holding Force In Pounds	Ratio of holding force to weight	Calculated Coefficient of Friction
5.9mm PowerCord	10.1		3.1	3.258	0.18 / .019
9mm Sterling HTP	10.3		4.5	2.289	0.13

Test Bar: .75" Diameter Aluminum Rack Bar					
Cordage / Rope	Weight Supported (in pounds)	2 Wrap Holding Force In Pounds	1 Wrap Holding Force In Pounds	Ratio of holding force to weight	Calculated Coefficient of Friction
5.9mm PowerCord	10.1		3.32	3.042	.17 / .18
5.9mm PowerCord	10.1	1.1		9.182	.17 / .18

Test Bar: 2.25" Diameter Aluminum Tube					
Cordage / Rope	Weight Supported (in pounds)	2 Wrap Holding Force In Pounds	1 Wrap Holding Force In Pounds	Ratio of holding force to weight	Calculated Coefficient of Friction
9mm Sterling HTP	22.3		9.02	2.472	.14 / .15
9mm Sterling HTP	22.3	3.96		5.631	.13 / .14

Test Bar: 2.25" Diameter Aluminum Tube

Cordage / Rope	Weight Supported (in pounds)	2 Wrap Holding Force In Pounds	1 Wrap Holding Force In Pounds	Ratio of holding force to weight	Calculated Coefficient of Friction
Petzl Cord 4.5 - 5mm	22.1		8.45	2.615	.15 / .16
Petzl Cord 4.5 - 5mm	22.1	3.4		6.500	0.15

Sandstone: 90 Degrees Of Contact Over Radiused Edge (90 Degrees = $\pi/2$)

Cordage / Rope	Weight Supported (in pounds)	Over The Edge Holding Force Force In Pounds	Ratio of holding force to weight	Calculated Coefficient of Friction
5.9mm Sterling PowerCord	22.2	7.35	3.020	0.70
9mm Sterling HTP	22.3	8.12	2.746	0.64
Petzl 4mm Cord	22.1	6.54	3.379	0.78
8.3mm Imlay Canyon Fire	22.2	7.60	2.921	0.68

Coefficient values and resulting forces:

Coefficient of Friction	1 wrap (360 degrees) Ratio to 1	2 wraps (720 degrees) Ratio to 1	3 wraps (1080 degrees) Ratio to 1	4 wraps (1440 degrees) Ratio to 1
0.05	1.369	1.874	2.566	3.513
0.06	1.457	2.125	3.098	4.517
0.07	1.552	2.410	3.741	5.808
0.08	1.653	2.732	4.517	7.467
0.09	1.760	3.098	5.454	9.601
0.10	1.874	3.513	6.586	12.345
0.11	1.996	3.984	7.952	15.872
0.12	2.125	4.517	9.601	20.408
0.13	2.263	5.122	11.593	26.239
0.14	2.410	5.808	13.998	33.736
0.15	2.566	6.586	16.902	43.376
0.16	2.732	7.467	20.408	55.770
0.17	2.909	8.467	24.641	71.705
0.18	3.098	9.601	29.752	92.193
0.19	3.299	10.887	35.924	118.536
0.20	3.513	12.340	43.376	152.406
0.21	3.741	13.998	52.373	195.953
0.22	3.984	15.872	63.237	251.943

0.23	4.242	17.998	76.355	323.930
0.24	4.517	20.408	92.193	416.488
0.25	4.810	23.140	111.310	535.491
0.26	5.122	26.239	134.408	688.498
0.27	5.454	29.752	162.289	885.223
0.28	5.808	33.736	195.953	1138.159
0.29	6.184	38.253	236.600	1463.366
0.30	6.586	43.376	285.678	1881.495
0.35	9.017	81.306	733.145	6610.796
0.40	12.345	152.406	1881.495	23227.599
0.45	16.902	285.678	4828.543	81612.160
0.50	23.140	535.491	12391.647	286751.313